TRINITY COLLEGE

Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

If required by your examination administrator, please place your student identification label in this box

Section Two: Calculator-assumed

WA student number: In figures | | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section Two: Calculator-assumed **65% (98 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

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Question 8 **(8 marks)**

A random sample of adults who were not working and not seeking work were recently asked for the main reason that they were not looking for work. The responses, categorised by the sex of the adult and their main reason, are summarised in the table below.

Reason	Male	Female
Education	143	116
Family considerations	33	117
Other	84	37

(a) How many adults gave a response? (1 mark)

(b) What percentage of the females gave education as their main reason? (2 marks)

(c) Construct a table showing column percentages for the above data, rounding entries to the nearest whole number. (3 marks)

Reason	Male	Female	
Education			
Family considerations			
Other			

(d) Discuss whether the data from the survey suggests the presence of an association between the variables sex and reason. (2 marks)

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Question 9 **(7 marks)**

(a) The monthly units of electricity u consumed by each apartment in a building was strongly associated with the average monthly maximum temperature, T °C. The least-squares line for the variables was $\hat{u} = 211.5 - 2.8T$.

(i) Predict the units of electricity consumed by an apartment in a month when the average monthly maximum temperature was 13°C. (1 mark)

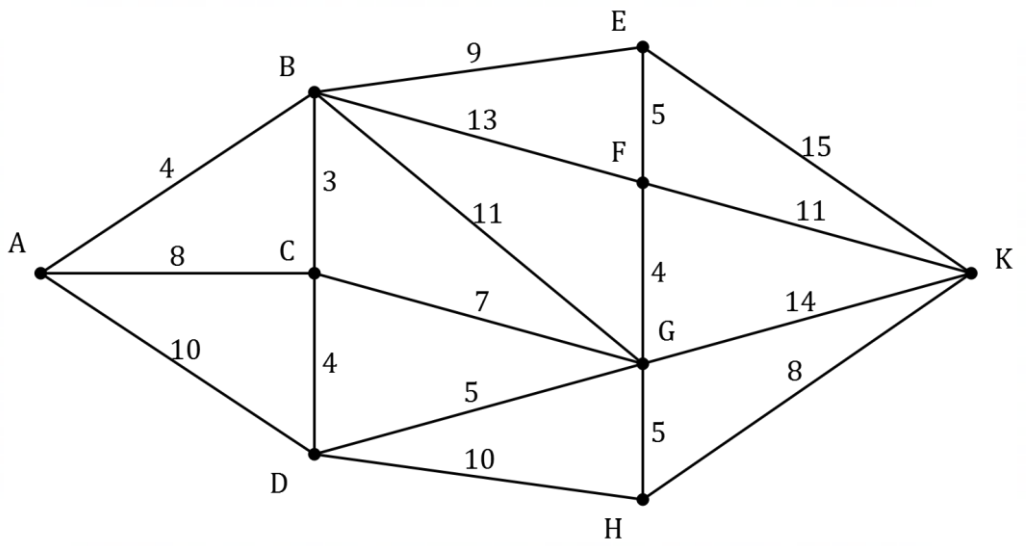
(ii) In a month when the average monthly maximum temperature was 24°C, an apartment consumed 146.6 units of electricity. Calculate the residual for this data point. (2 marks)

(b) In a government study, the correlation coefficient for the association between age and superannuation balance for employed adults was found to be 0.728. What percentage of the variation in superannuation balance for employed adults is unexplained by their variation in age? (2 marks)

(c) After measuring the age and hearing acuity of a group of pensioners, a researcher observed a negative linear association between the variables and found that 75% of the variation in hearing acuity can be explained by the variation in age. Determine the correlation coefficient for the association. (2 marks)

Question 10 **(7 marks)**

The graph below represents a network of distribution centres. Each edge weight is the cost in dollars to transport a parcel between adjacent centres (the vertices).



(a) Determine the minimum cost to transport a parcel from A to K and state the path that should be used to achieve this minimum. (3 marks)

(b) A new route is proposed between centres C and F which will reduce the minimum cost to transport a parcel from A to K by $\$4$. Determine the cost to transport a parcel between centres C and F. (2 marks)

(c) A parcel is transported along a route that is a cycle of 3 edges in the graph. Determine the maximum possible transport cost and describe the corresponding cycle. (2 marks)

See next page

Question 11 **(8 marks)**

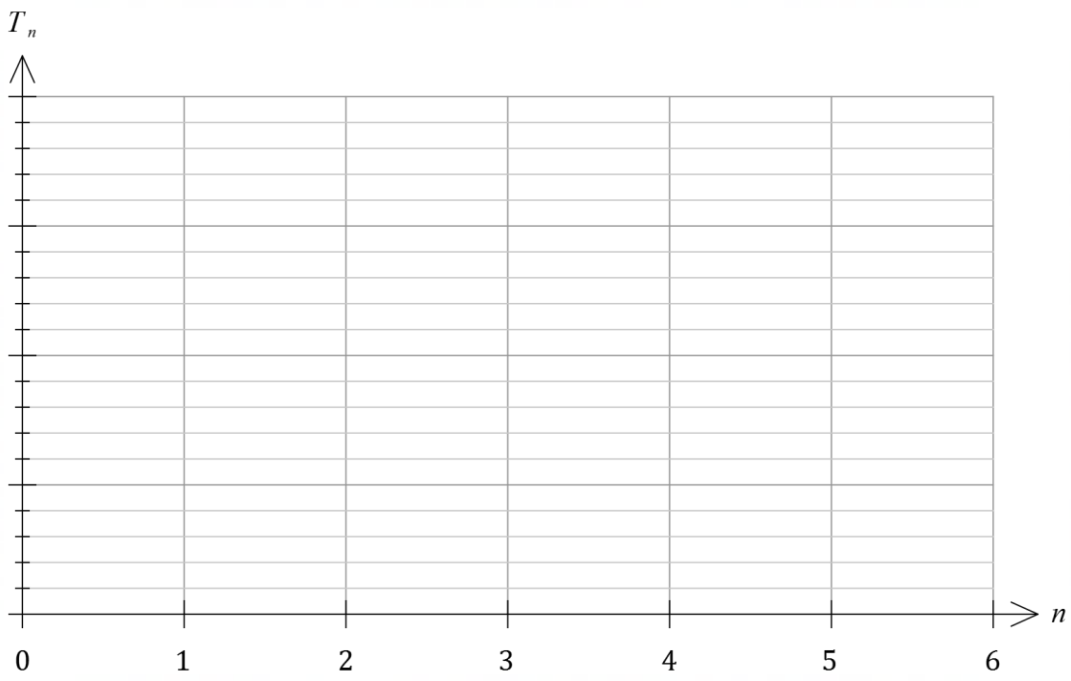
The cooling system for a mobile cool room has just been turned on. The temperature T_n °C inside the cool room, n hours later, is modelled by the linear recurrence relation

$$T_n = 0.68T_{n-1} + 1.12, \qquad T_0 = 19.$$

(a) Complete the table of temperatures below. (2 marks)

n	0	1	2	3	4	5	6
T_n (°C)	19.0	14.0					

(b) Add a scale to the vertical axis below and then plot the temperature inside the cool room every hour. (3 marks)



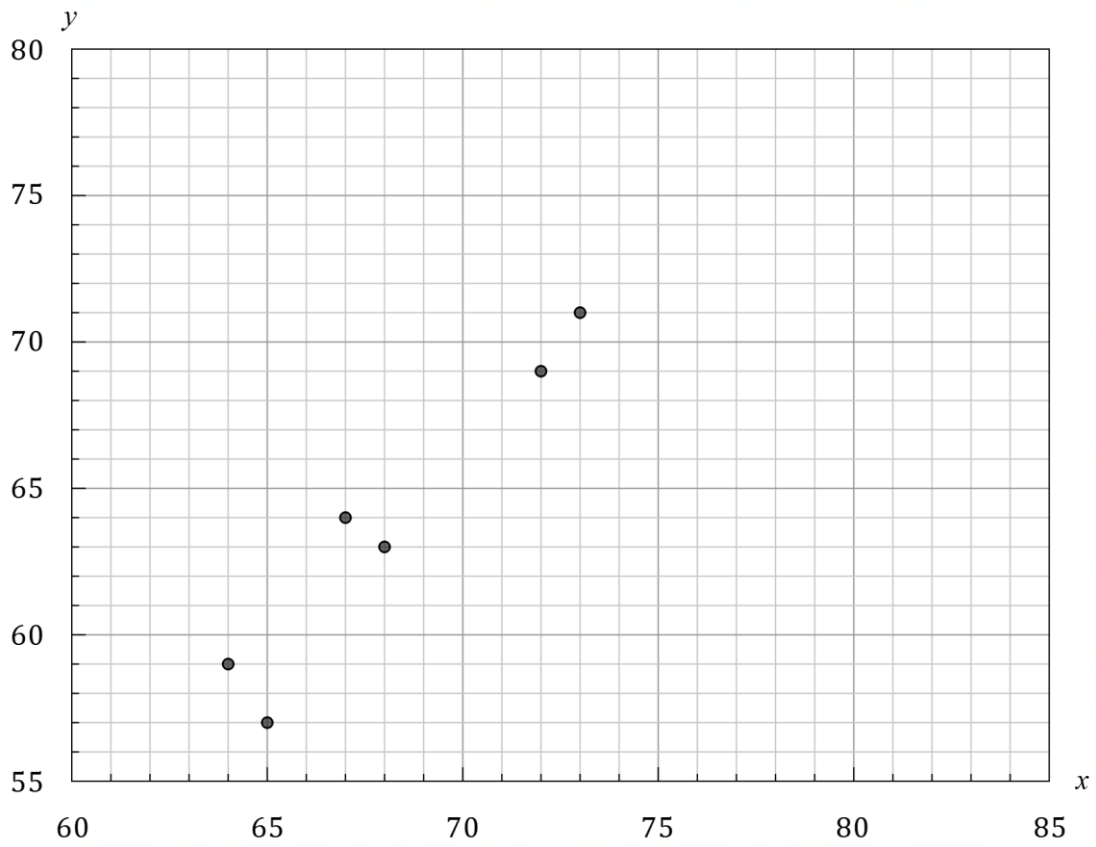
(c) After how many hours does the model predict that the temperature inside the cool room will first reach within $0.2°$ of its steady state? Justify your answer. (3 marks)

Question 12 **(13 marks)**

The table below shows the life expectancy, in years, of females and males in nine countries in Oceania.

Country	Female (x)	Male (y)
Federated States of Micronesia	72	69
Kiribati	64	59
Marshall Islands	73	71
Nauru	65	57
New Caledonia	80	74
Palau	78	68
Papua New Guinea	68	63
Solomon Islands	74	67
Tuvalu	67	64

(a) On the scatterplot below, plot the three missing data points that have been highlighted in the table. (2 marks)



(b) Determine the coefficient of determination between the variables and interpret its value in the context of the question. (2 marks)

See next page

(c) State the correlation coefficient between the variables and use its value to comment on the strength of the linear association between female and male life expectancy for these countries. (2 marks)

(d) Determine the equation of the least-squares line to model the relationship between the variables and draw this line on the scatterplot. (3 marks)

(e) The life expectancy of a female from Fiji is 70. Predict, to the nearest year, the life expectancy of a male from the same country and comment on any factors that affect the validity of your prediction. (2 marks)

(f) The life expectancy of a female Australian is 86. Predict, to the nearest year, the life expectancy of a male Australian and comment on any factors that affect the validity of your prediction. (2 marks)

See next page

Question 13 **(7 marks)**

Anna had a bank account that paid no interest. At the start of the year her account balance was $\$3450$, and at the end of the first week and every week thereafter she withdrew $\$75$.

(a) Calculate the balance of Anna's account after 4 weeks. (1 mark)

Let the balance in Anna's account at the end of the n^{th} week be A_n.

(b) Deduce a rule for A_n and hence determine the balance of Anna's account after 12 weeks. (2 marks)

(c) For how many weeks was Anna able to withdraw $\$75$? (1 mark)

In the same year, Anna's friend Ben had a similar account. The balance B_n of his account at the end of the n^{th} week was given by the recurrence relation $B_{n+1} = B_n - 60,\ B_0 = 3030$.

(d) Determine the balance of Ben's account after 12 weeks. (1 mark)

(e) At the end of one week during the year, the balance of Anna's account was identical to that of Ben's. Determine which week this was and the balance of both accounts at that time. (2 marks)

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Question 14 **(7 marks)**

A business bought a mainframe computer valued at $\$95\ 000$. The value of the computer depreciated by 35% each year.

(a) By how much did the value of the computer depreciate during the first year and what was its value one year after it was bought? (2 marks)

(b) Deduce a recursive rule for V_n, the value of the computer after n years. (2 marks)

(c) Calculate the value of the computer after 4 years. (1 mark)

(d) During which year does the value of the computer first depreciate by less than $\$1000$? Justify your answer. (2 marks)

Question 15 **(10 marks)**

An industrial chemist varied the amount of accelerant (a grams) used when making an epoxy resin and recorded the time taken (t seconds) for the resin to set. The results are shown below.

a	4.5	5.5	6.5	7.0	8.0	9.0	10.0	11.5	13.0	14.0
t	24.1	19.2	19.3	21.8	15.7	19.2	14.8	17.7	15.0	12.3

The chemist suspected that a linear association might exist between the variables and calculated the correlation coefficient $r_{at} = -0.81$.

(a) After seeing this value of the correlation coefficient, the chemist said to their assistant "it looks like there is a strong and negative linear association between the variables". Explain this interpretation of the coefficient. (2 marks)

The chemist also noted that the least-squares line for the data was $\hat{t} = 25.78 - 0.896a$ and used it to calculate nine residuals for the linear model as shown below, rounded to one decimal place.

a	4.5	5.5	6.5	7.0	8.0	9.0	10.0	11.5	13.0	14.0
Residual	2.3	-1.7	-1.7		-2.9	1.5	-2.0	2.2	0.9	-0.9

(b) Show how the residual of -2.9 was calculated and determine the residual associated with 7.0 grams of accelerant. (3 marks)

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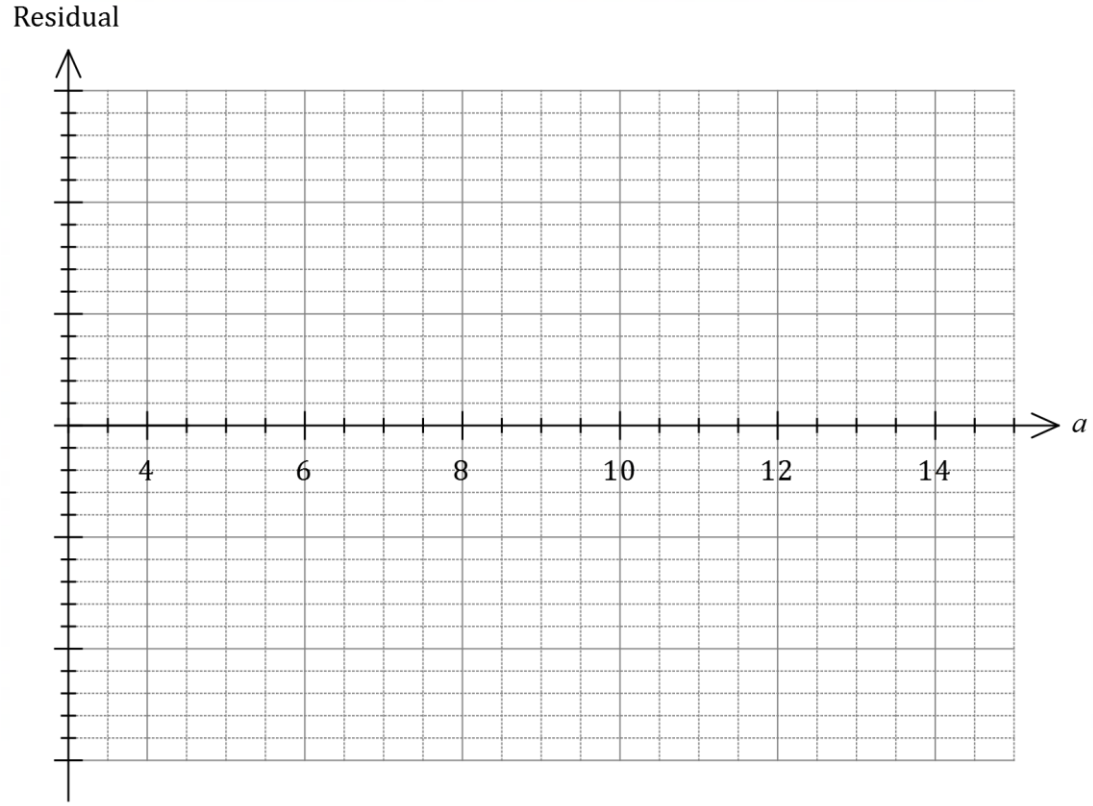
(c) Construct a residual plot for the data on the axes below. (3 marks)



(d) Does the residual plot support the chemist's suspicions that a linear model fits the data? Explain your answer. (2 marks)

Question 16 **(6 marks)**

The mass of a small puppy was measured as 625 g when it was one week old. A week later its mass had increased by 50 g.

(a) Assuming that the weekly mass of the puppy can be modelled by an arithmetic sequence, predict the mass of the puppy when it is 8 weeks old. (2 marks)

(b) Assuming that the weekly mass of the puppy can be modelled by a geometric sequence, predict the mass of the puppy when it is 8 weeks old. (3 marks)

(c) Comment on the usefulness of these models as the puppy gets older. (1 mark)

Question 17 **(8 marks)**

The annual number of mobile phone subscriptions and new cars sold in New South Wales, as collated by a researcher, are shown in the table below.

Year	2012	2013	2014	2015	2016	2017	2018
Subscriptions (s, in millions)	10.3	10.5	10.7	11.0	11.2	11.4	11.7
New cars (c, in thousands)	321	329	336	342	347	353	359

The researcher wanted to identify whether new car sales in New South Wales could be predicted from mobile phone subscriptions.

(a) Quantify the strength of the linear association between the variables s and c. (1 mark)

(b) Determine the equation of the least-squares line that can be used to predict c from s. (2 marks)

(c) Use the least-squares line to predict the number of new car sales in another Australian state that had 11.5 million mobile phone subscriptions, and comment, with reasons, on the validity of your prediction. (3 marks)

(d) Describe a possible non-causal explanation for the observed association between mobile phone subscriptions and new cars sold. (2 marks)

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Question 18 **(9 marks)**

The adjacency matrix for the connected planar graph P is $\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$.

(a) Determine, with justification, the number of faces that P has. (3 marks)

(b) Use elements from the adjacency matrix to explain why P is a simple graph. (3 marks)

(c) Ore's theorem states that a simple graph with n vertices is Hamiltonian if, for every pair of distinct vertices V_a and V_b which are not adjacent, the sum of the degrees of V_a and V_b is greater than or equal to n. Use Ore's theorem to show that P is Hamiltonian. (3 marks)

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Question 19 **(8 marks)**

Joe plans to invest $\$88\,000$ in an account that pays interest of 0.65% per month. At the end of each month, just after interest is added to the account, he will withdraw $\$330$. The balance of his account, a_n, after n withdrawals can be modelled by the recurrence relation

$$a_{n+1} = 1.0065a_n - 330, \qquad a_0 = 88\,000.$$

(a) Determine the balance of the account after 6 withdrawals have been made and describe how the balance has changed since the account was opened. (2 marks)

(b) Calculate the total withdrawn from the account after 6 withdrawals, and hence show that the total interest paid into the account over this time is $\$3455.80$. (2 marks)

(c) The balance of Joe's account will first exceed $\$95\,000$ after the k^{th} withdrawal. Determine the value of k and state the balance of the account at this time. (2 marks)

(d) If, after making the 6^{th} withdrawal, Joe then changed the amount he withdrew each month to $\$590$, explain how this would change the way the account balance grew in the future? Justify your answer. (2 marks)

End of questions

Supplementary page

Question number: _____